



# Journée « Mathématiques pour la neutronique » 2026

**Jeudi 2 Avril 2026**  
**Salle de séminaire du laboratoire LJLL**  
**Sorbonne Université, Paris**

**Barre 15-16, 3ème étage, porte 09**

La journée « Mathématiques pour la neutronique » est organisée conjointement entre le SERMA (François Madiot et Andrea Zoia), le RT Terre & Energies (Nicolas Seguin) et le laboratoire LJLL (Bruno Després), dans le cadre des activités du réseau thématique Terre & Énergies. Ce séminaire a pour thème l'application de techniques mathématiques avancées pour la solution de problèmes d'intérêt en physique des réacteurs nucléaires.

09h-9h15	Accueil - Café
09h15-9h30	Introduction
09h30-10h	Paul Rovel (CEA/DES) Méthodes Monte-Carlo pour la solution de l'équation du transport adjointe
10h-10h30	Olivier Hurisse (EDF) A random-choice scheme for scalar advection
10h30-10h45	Pause
10h45-11h15	Moreno Pintore (INRIA), Combining the Volume of Fluid method and neural networks: the 3D Volume of Fluid–Machine Learning method.
11h15-11h45	Axel Fauvel (CEA/DES), On the spectrum of the critical formulation of the linear transport operator
11h45-13h30	Pause déjeuner
13h30-14h	Pierre-Henri Tournier (LJLL), Simulation of Radiative Transfer in Variable 3D Atmosphere
14h-14h30	Gaël Poëtte (CEA/DAM), On the convergence of Monte Carlo -generalized Polynomial Chaos for uncertainty propagation in keff computations for neutronics
14h30-14h45	Pause
14h45-15h15	Thomas Collignon (CEA/DES), Extension d'un solveur de neutronique basé sur une formulation dG-Pn à des maillages 3D polyédriques comportant des faces planes et curvilignes
15h15-15h45	Alexis Anne (CEA/DES) Implémentation dans le code IDT d'une méthode de raffinement adaptatif de maillage (AMR) spatial pour la méthode des caractéristiques courtes dans le cas 3D.
15h45-16h15	Emiliano Masiello (CEA/DES) Positive Semidefinite Transport Correction to Prevent Negative $P_N$ Scattering Source Contributions in Discrete-Ordinates Simulations
16h15-16h30	Conclusion

## ABSTRACTS

**Paul Rovel (CEA/DES)**

*Méthodes Monte-Carlo pour la solution de l'équation du transport adjointe*

Dans des calculs Monte Carlo à source fixe, si le détecteur est petit, il peut être complexe d'obtenir suffisamment de contributions dans la zone d'intérêt et donc d'obtenir une bonne variance.

Le principe du transport adjoint consiste à échantillonner des particules qui sont déjà arrivées dans la zone du détecteur et échantillonner leurs histoires «à rebours» en se demandant comment la particule a pu arriver dans l'état actuel. L'histoire se construit donc en partant de la fin en ajoutant progressivement les vols et collisions qui ont mené la particule dans son état. Si la particule provient effectivement de la source, une contribution est décomptée. Cette méthode permet d'obtenir le même résultat qu'avec les méthodes Monte Carlo habituelles ("directes") mais potentiellement avec une variance plus faible si la taille dans l'espace des phases de la source est plus grande que celle du détecteur (ce qui est le cas dans de nombreux cas de radioprotection où la source est le cœur ou un groupe d'assemblages et le détecteur peut être un dosimètre ou encore un opérateur).

L'échantillonnage «à rebours» des collisions et des vols pose cependant des problèmes. Les données nucléaires étant présentées dans le sens «direct», il faut les «renverser» et préparer des données adjointes qui permettront d'échantillonner à rebours. De plus, contrairement au transport direct, les particules adjointes n'ont pas de «réalité physique», ce qui se manifeste par un opérateur adjoint bien moins approprié aux Méthodes Monte-Carlo que le cas direct. La conséquence en est que le processus d'échantillonnage à rebours implique de nombreuses corrections de poids qui ont des effets sur la répartition des poids de la population de particules simulées et donc sur la stabilité du calcul, il est donc nécessaire de comprendre ces phénomènes et de contrôler les poids pour obtenir une variance acceptable.

**Olivier Hurisse (EDF)**

*A random-choice scheme for scalar advection*

This talk is dedicated to a numerical method based on a random choice as proposed in Glimm's scheme. It is applied to the problem of advection of a scalar quantity. The numerical scheme proposed here relies on a fractional step approach for which: the first step is performed using any classical finite-volume scheme, and the second step is a cell-wise update. This second step is a projection based on a random choice. The resulting scheme possesses a very low level of numerical diffusion. In order to assess the capabilities of this approach, several test cases have been investigated including convergence studies with respect to the mesh-size. The algorithm performs very well on one-dimensional and multi-dimensional problems. This algorithm is very easy to implement even for multi-processor computations.

**Moreno Pintore (INRIA)**

*Combining the Volume of Fluid method and neural networks: the 3D Volume of Fluid–Machine Learning method*

In this talk, we present the three-dimensional extension of the Volume of Fluid–Machine Learning (VOF-ML) method. The VOF-ML is a hybrid approach, originally proposed in [1] for two-dimensional problems, that combines the robustness and accuracy of the Volume of Fluid (VOF) method with the nonlinear approximation capabilities of neural networks.

The term VOF refers to a class of methods used to advect sharp interfaces between different materials. It is mainly employed in combination with other finite volume schemes to ensure exact conservation of the physical quantities of interest. In the VOF framework, the numerical flux is computed using the volume

fractions (i.e., the proportions of each material in the cells) within each cell at a given time step. In most cases, updating the volume fractions at the next time step requires an interface reconstruction procedure. However, this reconstruction may lead to excessive diffusion or artificial artifacts, particularly when the exact interface is non-smooth or characterized by regions with high curvature.

In contrast, the VOF-ML scheme approximates the flux directly from the volume fractions using a neural network, without reconstructing the local interface. The method was introduced in [1] and later extended to problems involving more than two materials in [2], but it remained limited to two-dimensional settings. In this talk, we focus on its extension to three-dimensional problems and the associated implementation challenges. In particular, special attention must be paid to the generation of the training dataset and to the enforcement of physical constraints. Numerical tests are presented to assess the performance of the proposed scheme, showing that the optimal convergence rate with respect to mesh refinement is achieved.

[1] Bruno Després and Hervé Jourdain. Machine learning design of volume of fluid schemes for compressible flows. *Journal of Computational Physics*, 408:109275, 2020.

[2] Matthieu Ancellin, Bruno Després, and Stéphane Jaouen. Extension of generic two-component vof interface advection schemes to an arbitrary number of components. *Journal of Computational Physics*, 473:111721, 2023.

**Axel Fauvel** (CEA/DES)

*On the spectrum of the critical formulation of the linear transport operator*

The spectrum of the linear transport operator has been thoroughly investigated in the form that came to be known as  $\alpha$ -eigenvalues. Rather surprisingly, the alternative formulation known as  $k$ -eigenvalue problem, which is widely used in reactor physics applications, has been much less thoroughly characterized: the theoretical knowledge of its spectrum is fragmented and incomplete.

In this work, we gather some existing and little-known findings available in the literature concerning the  $k$ -eigenvalue problem. We improve and extend the existing proofs to broader settings within a continuous-energy framework, as much as possible without assuming any specific form of the kernels of the Boltzmann equation. Both the  $L_2$  case, which by interpolation yields the  $L_p$  case,  $1 < p < \infty$ , and the  $L_1$  case are investigated. Under proper assumptions, we show that the operator associated to the  $k$ -eigenvalue problem is compact and that its spectrum is independent of the chosen Lebesgue space. Under more restrictive conditions, we exhibit the existence, uniqueness and simplicity of a positive dominant eigenvector, and establish the existence of a spectral gap, ensuring convergence of the power iteration method. The present findings are preliminary, in that the requirements for the statements concerning the  $k$ -eigenvalue spectrum to hold true might be further relaxed in the future.

**Pierre-Henri Tournier** (LJLL)

*Simulation of Radiative Transfer in Variable 3D Atmosphere*

The radiative transfer equations have three spatial dimensions, two angular dimensions describing the radiation direction, one frequency variable, and possibly time. In this work, without loss of generality, they are reduced to a nonlinear integro-differential system and solved by an iterative scheme which is shown to be monotone and convergent. At each step, two large integrals of convolution type need to be computed. The kernel matrices are assembled in compressed form using Hierarchical matrices, with the help of the H-Matrix high performance parallel library Htool [1]. The whole method is implemented in FreeFEM [2], a free open-source software designed for multi-physics simulations. The resulting method is of complexity  $n \log(n)$  where  $n$  is the number of vertices of the physical domain. As an example, the method is applied to the temperature and radiation in the valley of Chamonix (France), the challenge being the large variation of density, altitude and light reflection due to snow and clouds. Our method is able to handle 240K physical points, all directions of radiation and 683 frequencies in less than 35 minutes on an Apple M1 laptop [3]. The method has also been extended to handle reflective boundary conditions [4].

[1] Marchand, P., Tournier, P. H., & Jolivet, P. (2026). Htool-DDM: A C++ library for parallel solvers and compressed linear systems. *Journal of Open Source Software*, 11(118), 9279

- [2] Hecht, F. (2012). New development in FreeFem++. Journal of numerical mathematics, 20(3-4), 1-14
- [3] Golse, F., Hecht, F., Pironneau, O., Smets, D., & Tournier, P. H. (2023). Radiative transfer for variable three-dimensional atmospheres. Journal of Computational Physics, 475, 111864
- [4] Pironneau, O., & Tournier, P. H. (2023). Reflective conditions for radiative transfer in integral form with h-matrices. Journal of Computational Physics, 495, 112531

### **Gaël Poëtte** (CEA/DAM)

*On the convergence of Monte Carlo -generalized Polynomial Chaos for uncertainty propagation in keff computations for neutronics*

This talk addresses the analysis of power iteration methods applied to eigenvalue problems involving uncertainty. While several uncertain power iteration procedures have been proposed in the literature and have demonstrated empirical convergence, a rigorous theoretical justification of their convergence properties remains lacking. The objective of this work is to investigate the convergence behavior of such methods in a stochastic or parametric uncertainty setting. The proposed framework focuses on the propagation of uncertainty through the MC-gPC [1, 2] power iteration process and on the characterization of the resulting random eigenvalues and eigenfunctions. By combining spectral analysis with probabilistic tools, the study establishes conditions under which convergence of the P-truncated gPC approximation of the eigenvalues and eigenvectors can be guaranteed.

- [1] G. Poëtte, E. Brun, Efficient uncertain k eff computations with the Monte Carlo resolution of generalised Polynomial Chaos Based reduced models, J. Comp. Phys. (Nov. 2020).
- [2] T. Bonnet, A. Dubey, E. Shwageraus, Addressing geometrical perturbations by applying generalized polynomial chaos to virtual density in continuous energy monte-carlo power iteration (2025). arXiv:2506.06046.

### **Thomas Collignon** (CEA/DES)

*Extension d'un solveur de neutronique basé sur une formulation dG-Pn à des maillages 3D polyédriques comportant des faces planes et curvilignes*

Cette présentation porte sur l'extension du solveur de neutronique NYMO, basé sur une formulation dG-Pn, à des maillages 3D polyédriques comportant des faces planes et curvilignes. Les travaux présentés se concentrent sur le calcul des matrices élémentaires nécessaires à la résolution de l'équation de transport neutronique stationnaire. Cette approche permet non seulement de modéliser les géométries complexes rencontrées dans les réacteurs innovants, mais aussi d'éviter l'étape d'homogénéisation lors de la résolution pour gagner en précision.

### **Alexis Anne** (CEA/DES)

*Implémentation dans le code IDT d'une méthode de raffinement adaptatif de maillage (AMR) spatial pour la méthode des caractéristiques courtes dans le cas 3D*

La résolution de l'équation du transport neutronique nécessite le traitement d'un grand nombre de degrés de liberté. Nous présentons ici une méthode de raffinement adaptatif de maillage (AMR) spatial appliquée à la méthode des caractéristiques courtes. Cette méthode est intégrée au solveur IDT du code APOLLO3®. Au cours de cette présentation, nous donnerons une brève description de l'analyse de l'erreur qui sous-tend l'AMR, afin de nous concentrer principalement sur son implémentation dans le solveur IDT ainsi que sur les cas tests étudiés. Nous détaillerons ensuite l'algorithme d'AMR et, en particulier, la structure du maillage en octree et le nouvel algorithme de balayage nécessaire à la fois au maillage non conforme et à l'AMR. Cette méthode permettrait, après une implémentation plus optimisée, un gain en mémoire et présente d'ores et déjà des temps de calcul compétitifs par rapport au balayage classique d'IDT. Les résultats seront présentés en géométrie 3D à l'aide des benchmarks de Kobayashi.

**Emiliano Masiello (CEA/DES)**

*Positive Semidefinite Transport Correction to Prevent Negative  $P_N$  Scattering Source Contributions in Discrete-Ordinates Simulations*

One of the numerical issues of discrete-ordinates transport simulations is the emerging of pathological negative scattering source. This problem produces negative scalar fluxes despite the positivity of the spatial scheme. The causes behind such undesired effect are particularly linked to the truncation error emerging from the finite  $P_N$  expansion of the scattering cross section. Such an error is pronounced in highly anisotropic materials when used in physical problems characterized by a particularly peaked angular distribution of the flux. First, materials having forward or backward peaking scattering could lead to truncated  $P_N$  expansions that leads to negative values in some ranges of the scattering cosine. Secondly, in radiation protection problem, the angular flux of target regions, which are far from the point-source, shapes as a Dirac delta function centered in the direction linking the target point to the source. In these regions, the angular moments of the flux are all equal to each other, favoring, thus, the appearance of negative scattering contributions in some  $S_N$  direction. This could bring to a negative scalar flux annoying particularly  $S_N$  simulations when used in the framework of variance reduction technique, as CADIS or FW-CADIS. In these techniques, employed to enhance Monte Carlo simulations, the solution must be guaranteed to be positive so as not to make particles' weights during the Monte Carlo simulation meaningless.

In this work, we present the Positive SemiDefinite Transport Correction (PSD-TC) that modifies the scattering cross section as well as the total cross section to guarantee the positivity of the scattering contributions over the unit spheres. We show an application of the PSD-TC to  $P_1$  and  $P_2$  expansions. We will present the results comparing the technique to the uncorrected kernel simulations and we will measure as well its effectiveness both in terms of positivity of the solution and of iterative/runtime performances.