Journée « Mathématiques pour la neutronique » 2025

Jeudi 27 Mars 2025
Salle de séminaire du laboratoire LJLL
Sorbonne Université, Paris

Barre 15-16, 3ème étage, porte 09

La journée « Mathématiques pour la neutronique » est organisée conjointement entre le SERMA (François Madiot et Andrea Zoia), le RT Terre & Energies (Nicolas Seguin) et le laboratoire LJLL (Bruno Després), dans le cadre des activités du réseau thématique Terre & Énergies.

Ce séminaire a pour thème l'application de techniques mathématiques avancées pour la solution de problèmes d'intérêt en physique des réacteurs nucléaires.

09h45-10h	Accueil - Café
10h-10h30	Cecilia Montecchio (CEA/DES), Revisiting the reactor period capabilities in TRIPOLI-4®
10h30-11h	Matthias Schlottbom (University of Twente), On accelerated iterative schemes for neutron transport using residual minimization
11h-11h30	Axel Fauvel (CEA/DES), A Monte Carlo strategy for the neutron noise equation without linearization – Concept and first results
11h30-13h30	Pause déjeuner
13h30-14h	Matt Evans (Bath University), Cycle-Free Polytopal Mesh Sweeping for Boltzmann Transport
14h-14h30	Gérald Samba (CEA/DAM), Generalized Levermore-Pomraning equations with memory effects
14h30-15h	Daniel Yaacoub (Université Clermont-Auvergne), Branching Stochastic Processes for path-integral representation of photosynthetic charges drift-diffusion
15h-15h15	Pause café
15h15-15h45	Ruggero Rosselli (CEA/DES), Adjoint Sensitivity Methods for the Multi-Physics of Molten Salt Reactors
15h45-16h15	Axel Laureau (CNRS), From Monte Carlo simulation to the use of transfer functions for multiphysics coupling
16h15-16h45	Thayz Gomes Ferreira (CEA/DES), Zero-variance Monte Carlo games for radiation shielding problems including neutron multiplication
16h45-16h55	Conclusion

ABSTRACTS

Cecilia Montecchio (CEA/DES)

Revisiting the reactor period capabilities in TRIPOLI-4®

Reactivity measurements in reactor analysis require the assessment of the long-time behaviour of the system under consideration. For this purpose, it is customary to resort to the α-eigenvalue formulation of the Boltzmann equation: it is well-known that the dominant α eigenvalue corresponds to the reciprocal of the asymptotic reactor period. Proving the well-posedness of the α-eigenvalue equation for arbitrary reactor configurations (and associated cross sections) is a non-trivial task; yet, for most systems of practical relevance the existence of a dominant α eigenvalue can be safely assumed [1]. The considerable significance of α as a measurable quantity, as opposed to the k eigenvalue, has fostered the development of several Monte Carlo methods to accurately determine this parameter. The most common approach to solve the α-eigenvalue equation, known as α-k iteration approach, is a modified version of the standard power iteration method traditionally used for k eigenvalue calculations, where the α eigenvalue is treated as an adjustable parameter; various implementations have been attempted, based on modified collision events [2, 3] or on modified flight events [4]. Strategies trying to estimate α directly, without any adjustment procedure based on a fictitious k eigenvalue, have been also conceived, such as the source method [5] and the α-tally method: their underling idea is to express α as the dominant root of a polynomial whose coefficients can be estimated by Monte Carlo as reaction rates using a modified power iteration [6, 7]. The legacy algorithm for reactor period calculations in TRIPOLI-4®, the Monte Carlo code developed at CEA, is the α-k method. Prompted by the findings of Ref. 6, a tally-based strategy has been implemented with several variants. Within this framework, the benefits and drawbacks of all the approaches tested are demonstrated using a selection of relevant benchmark configurations.

References

[1] G. I. Bell and S. Glasstone. Nuclear Reactor Theory. Van Nostrand and Reinhold, NY, USA, 1970.
[2] Andrea Zoia, Emeric Brun, and Fausto Malvagi. α eigenvalue calculations with TRIPOLI-4®. Annals of Nuclear Energy, 63:276–284, 2014.
[3] D. Brockway, P. Soran, and P. Whalen. Monte Carlo α Calculation. Technical Report LA-UR-85-1224, Los Alamos National Laboratory (LANL), Los Alamos, NM (United States), 1985.
[4] T. Yamamoto and Y. Miyoshi. An algorithm of α- and γ-mode eigenvalue calculations by Monte Carlo method. In Proc. 7th ICNC. Japan Atomic Energy Research Institute (JAERI), 2003.
[5] H. J. Shim et al. Monte Carlo Alpha Iteration Algorithm for a Prompt Neutron Decay Constant Calculation. Transactions of the American Nuclear Society, 111:725–726, 2014.
[6] Colin James Josey and Forrest B. Brown. A New Monte Carlo α-Eigenvalue Estimator with Delayed Neutrons. Transactions of the American Nuclear Society, 118:903–906, 2018. [7] Yasushi Nauchi. Attempt to Estimate Reactor Period by Natural Mode Eigenvalue Calculation. In Proc. of SNA + MC, Paris, France, 2013.

Matthias Schlottbom (University of Twente)

On accelerated iterative schemes for neutron transport using residual minimization

The numerical solution of neutron transport problems requires the solution of very large linear systems.
In the past decades a vast amount of iterative schemes has been devised to solve this task.
To accelerate convergence of such iterative methods, preconditioners have been developed that solve a diffusion problem, which can be well motivated using arguments from asymptotic analysis. It has been observed that special care needs to be taken in the discretization of such diffusion problems to preserve convergence, leading to so-called consistent schemes.
In this talk, we take a slightly different point of view and use preconditioners that are based on residual minimization over suitable subspaces.

We prove convergence of the resulting iteration using Hilbert space norms, which allows us to obtain algorithms that converge robustly with respect to finite dimensional realizations via Galerkin projections. We investigate in particular the behavior of the iterative scheme for discontinuous Galerkin discretizations in the angular variable in combination with subspaces that are derived from related diffusion problems. The performance of the resulting schemes is investigated in numerical examples for highly anisotropic scattering problems with heterogeneous parameters.

Gérald Samba (CEA/DAM)

Generalized Levermore-Pomraning equations with memory effects

The Levermore-Pomraning (LP) equations allow obtaining approximate solutions to particle transport problems in Markov media, reducing the computation time compared to reference solutions resulting from sampling a collection of material realizations and solving for each of them the linear transport equation. The deterministic LP equations have a stochastic counterpart, the Chord Length Sampling (CLS) model, which can be easily implemented in a Monte Carlo particle-transport solver with minimal modifications. It is known that significant discrepancies may appear between exact and LP (or CLS) solutions, because of the memory-less nature of the LP (or CLS) model. Over the years, several CLS-like improved models have been proposed: greater accuracy with respect to reference solutions has been achieved by adding `spatial memory' effects to the stochastic particle-transport algorithms. In this work, we present novel enhanced-memory LP model in one-dimensional geometry, obtained by introducing a mathematical formalism to take into account memory effects in the LP equations. We will benchmark our results against the standard LP and CLS models, the improved CLS-like models, and the reference solutions for a set of spatially heterogeneous configurations.

Axel Fauvel (CEA/DES)

A Monte Carlo strategy for the neutron noise equation without linearization – Concept and first results

Neutron noise describe the fluctuation of the neutron flux around its mean value induced by core perturbations. Usual linear noise theory assume a small enough perturbation to neglect terms containing the product of perturbed quantities in the underlying Fourier-transformed Boltzmann equation. However, in case of interest such as mechanical vibration of fuel pins, this hypothesis is not satisfied. We propose a Monte Carlo strategy to solve the noise equation in the frequency domain in a non-perturbative approach, i.e. without resorting to linearization. A proof of concept is carried out on simple case, revealing this method’s potential, but raising some convergence issues to be discussed.

Matt Evans (Bath University)

Cycle-Free Polytopal Mesh Sweeping for Boltzmann Transport

We introduce a novel property of bounded Voronoi tessellations that enables cycle-free mesh sweeping algorithms. We prove that a topological sort of the dual graph of any Voronoi tessellation is feasible in any flow direction and dimension, allowing straightforward application to discontinuous Galerkin (DG) discretisations of first-order hyperbolic partial differential equations and the Boltzmann Transport Equation (BTE) without requiring flux-cycle corrections.
We also present an efficient algorithm to perform the topological sort on the dual mesh nodes, ensuring a valid sweep ordering. This result expands the applicability of DG methods for transport problems on polytopal meshes by providing a robust framework for scalable, parallelised solutions. To illustrate its effectiveness, we conduct a series of computational experiments showcasing a DG scheme for BTE, demonstrating both computational efficiency and adaptability to complex geometries.

Daniel Yaacoub (Université Clermont-Auvergne)

Branching stochastic processes for Feynman-Kac representations of drift-involving non-linearities

Probabilistic representations of non-linear physics have been enabled -until recent advances- by step forward approaches extending Feynman-Kac theory, initially based on superposition and linearity, to a first class non-linear physics. This has resulted in reactive nonlinearities, benefiting from a conceptual framework with a unique process propagating toward sources, so-called branching stochastic process. For non-linearities involving drift-velocity, such a feat has not yet been achieved. Yet large and diverse theoretical and applied communities are concerned: drift-diffusion transport models including Navier-Stokes, Poisson-Nernst-Planck or Keller-Segel equations, rely on a coupling with a sub-model of the drift velocity.

An extension of Feynman-Kac representations to such drift-diffusion phenomenologies coupled to a model of the drif field in confined spaces is proposed. This extension relies on the ability to construct a continuous branching advecto-diffusive process for which each branch carries information about the drift field. In this way, we allow probabilistic and propagative pictures of such physics in a single branching path-space, as previously developed through the use of branching Brownian motions, Galton-Watson processes or growth-fragmentation processes for reactive non-linearities (KPP, Boltzmann) in neutronics.

The first expected consequence is to provide renewed insights in terms of path-space propagative pictures. The second is an alignment with applied mathematics, computer graphics, and engineering communities addressing complex geometries. Hence, benefiting of computer science acceleration techniques in making the computational time insensitive to the geometric refinement while statistically sampling such path-space using Monte Carlo methods, numerical implementations applied to electron transport, coupled with a model of the electric field, are finally handled in a porous photoanode.

Ruggero Rosselli (CEA/DES)

Adjoint Sensitivity Methods for the Multi-Physics of Molten Salt Reactors

Molten Salt Reactors (MSRs) are a promising technology for the future of nuclear energy. The complex multi-physics of MSRs requires the development of advanced neutronic and thermalhydraulic coupling models. Sensitivity analysis is a powerful tool to study the impact of input parameters on the system’s responses. Adjoint sensitivity methods are particularly well-suited for complex systems with many input parameters and few output responses, such as reactor physics problems.
This work presents the adjoint sensitivity framework for a 1D model of the neutronics – thermohydraulics coupled MSR, using the Lagrangian formalism. The model is implemented numerically in Python, and some results are discussed using the Molten Salt Fast Reactor data. The adjoint sensitivities calculated are in good agreement with finite differences, showing the potential of adjoint methods for the multi-physics of MSRs.

Axel Laureau (CNRS)

From Monte Carlo simulation to the use of transfer functions for multiphysics coupling

Monte Carlo simulations offer an accurate estimation of the solution to the Boltzmann equation, however at a high computational cost. This presentation explores an approach allowing to reduce significantly this computational cost in situations where numerous evaluations are required, such as in multiphysics coupling for instance. The idea relies on the precomputation transfer functions obtained by Monte Carlo, combined to perturbative methods allowing to take into account variations in the media efficiently.

Thayz Gomes Ferreira (CEA/DES)

Zero-variance Monte Carlo games for radiation shielding problems including neutron multiplication

Zero-variance sampling strategies are an ideal class of Monte Carlo games capable of estimating a given response function without uncertainty. Such games cannot be used in practice, since their implementation requires the exact knowledge of the adjoint flux, but they provide the theoretical foundations to guide effective importance-sampling schemes such as the Consistent Adjoint-Driven Importance Sampling (CADIS) strategy, which is nowadays available in most Monte Carlo simulation codes. In this work we explore the behaviour of zero-variance Monte Carlo games where particle histories are allowed to branch at collision events, in the context of radiation shielding problem with neutron multiplication. Branches are

typically occurring at fission events. Such ‘branching’ games are contrasted to ‘branchless’ zero-variance strategies: using a few relevant benchmark configurations, we show that branchless games generally perform better in terms of computing time and variance of the sought tally.